

# The Effect of Material Model Formulation in the Stress Analysis of Abdominal Aortic Aneurysms

JOSE F. RODRÍGUEZ,<sup>1</sup> GIAMPALO MARTUFI,<sup>2</sup> MANUEL DOBLARÉ,<sup>1</sup> and ENDER A. FINOL<sup>3</sup>

<sup>1</sup>Group of Structural Mechanics and Materials Modeling, Aragon Institute of Engineering Research (I3A) Torres Quevedo Building, María de Luna 3, Zaragoza 50018, Spain; <sup>2</sup>Royal Institute of Technology, Department of Solid Mechanics, 100 44 Stockholm, Sweden; and <sup>3</sup>Institute for Complex Engineered Systems, Biomedical Engineering Department and Department of Mechanical Engineering, Carnegie Mellon University, 5000 Forbes Avenue, 1205 Hamburg Hall, Pittsburgh, PA 15213-3890, USA

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**Abstract**—A reliable estimation of wall stress in Abdominal Aortic Aneurysms (AAAs), requires performing an accurate three-dimensional reconstruction of the medical image-based native geometry and modeling an appropriate constitutive law for the aneurysmal tissue material characterization. A recent study on the biaxial mechanical behavior of human AAA tissue specimens demonstrates that aneurysmal tissue behaves mechanically anisotropic. Results shown in this communication show that the peak wall stress is highly sensitive to the anisotropic model used for the stress analysis. In addition, the present investigation indicates that structural parameters (e.g., collagen fiber orientation) should be determined independently and not by means of non-linear fitting to stress–strain test data. Fiber orientation identified in this manner could lead to overestimated peak wall stresses.

**Keywords**—Aneurysm, Wall stress, Anisotropy, Soft tissue mechanics, Computational modeling, Rupture, Biomechanics, Thrombus.

## INTRODUCTION

In biomechanical terms, AAA rupture is a phenomenon that occurs when the developing mechanical stresses within the aneurysm inner wall, because of the exerted intraluminal pressure, exceed the failure strength of the aortic tissue. Recent studies show that peak wall stress in AAAs is a reliable parameter to assess rupture potential. Fillinger *et al.*<sup>3</sup> performed in vivo analysis of mechanical wall stress and AAA rupture risk, and showed that peak wall stress is a more reliable parameter than maximum transverse diameter in predicting rupture potential. To obtain a reliable

estimation of wall stress, it is necessary to perform an accurate three-dimensional reconstruction of the AAA geometry and identify an appropriate constitutive law for the aneurysmal tissue. In this regard, physiologic and biomechanical studies show that the AAA wall is a heterogeneous material undergoing large strains prior to failure<sup>7</sup> and deforming in an isochoric manner.<sup>2</sup> In addition to these observations, a recent study on the biaxial mechanical behavior of human AAA tissue specimens<sup>15</sup> demonstrates that aortic aneurysmal tissue behaves mechanically anisotropic. In this work, we perform finite element stress analysis in five patient specific AAA models using different anisotropic constitutive models with the objective of investigating the sensitivity of wall stress to population-averaged material model parameters.

## METHODS

We consider the aneurysmal wall as a hyperelastic material and postulate the existence of a strain-energy function (SEF)  $W$ , from which the stress–strain behavior of the material can be derived.<sup>5</sup> In this investigation, isotropic and anisotropic constitutive equations were used to model the mechanical behavior of the aneurysmal wall. For the case of isotropy, the material response of the aneurysm is characterized by the SEF<sup>11</sup>

$$W_{\text{WALL,iso}}(\mathbf{C}) = U(J) + c_{10}(\bar{I}_1 - 3) + c_{20}(\bar{I}_1 - 3)^2, \quad (1)$$

where  $\bar{I}_1 = \text{tr}\bar{\mathbf{C}}$ , and  $c_{10}$  and  $c_{20}$  are constants with dimension of stress. For the case of anisotropy, the aneurysmal tissue was modeled as a hyperelastic material reinforced with two families of fibers aligned along two directions arranged in a double-helix

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Address correspondence to Ender A. Finol, Institute for Complex Engineered Systems, Biomedical Engineering Department and Department of Mechanical Engineering, Carnegie Mellon University, 5000 Forbes Avenue, 1205 Hamburg Hall, Pittsburgh, PA 15213-3890, USA. Electronic mail: finole@cmu.edu

pattern.<sup>6</sup> Two different models were used to describe the anisotropic behavior of the arterial wall. The classical framework proposed by Holzapfel *et al.*<sup>6</sup> is used as the first anisotropic model, i.e.,

$$W_{\text{WALL,anisol}}(\mathbf{C}) = U(J) + c_{10}(\bar{I}_1 - 3) + \frac{k_1}{2k_2} \left[ e^{k_2(\bar{I}_4 - 1)^2} - 1.0 \right] + \frac{k_3}{2k_4} \left[ e^{k_4(\bar{I}_6 - 1)^2} - 1.0 \right], \quad (2)$$

where  $\bar{I}_4 = \mathbf{n}_0 \cdot \bar{\mathbf{C}} \cdot \mathbf{n}_0$ ,  $\bar{I}_6 = \mathbf{m}_0 \cdot \bar{\mathbf{C}} \cdot \mathbf{m}_0$ ,  $c_{10}$ , and  $k_1 \dots k_4$  are material parameters. In addition, it is assumed that the anisotropic terms only contribute when either  $\bar{I}_4 > 1$  or  $\bar{I}_6 > 1$ . The second anisotropic model used corresponds to that proposed by Rodríguez *et al.*<sup>13</sup>:

$$W_{\text{WALL,anisot2}}(\mathbf{C}) = U(J) + c_{10}(\bar{I}_1 - 3) + \frac{k_1}{2k_2} \left\{ e^{k_2[(1-\rho)(\bar{I}_1 - 3)^2 + \rho(\bar{I}_4 - \bar{I}_4^0)^2]} - 1.0 \right\} + \frac{k_3}{2k_4} \left\{ e^{k_4[(1-\rho)(\bar{I}_1 - 3)^2 + \rho(\bar{I}_6 - \bar{I}_6^0)^2]} - 1.0 \right\}, \quad (3)$$

where  $\bar{I}_4^0$  and  $\bar{I}_6^0$  are additional parameters related to fiber crimping and  $\rho$  is a (dimensionless) measure of anisotropy. Since the presence of intraluminal thrombus (ILT) redistributes the stress at the aneurysm wall, ILT was included and modeled, to the extent present in all aneurysm sacs, as an isotropic hyperelastic material with the SEF<sup>1</sup>

$$W_{\text{ILT,iso}}(\mathbf{C}) = U(J) + c_{01}(\bar{I}_2 - 3) + c_{02}(\bar{I}_2 - 3)^2, \quad (4)$$

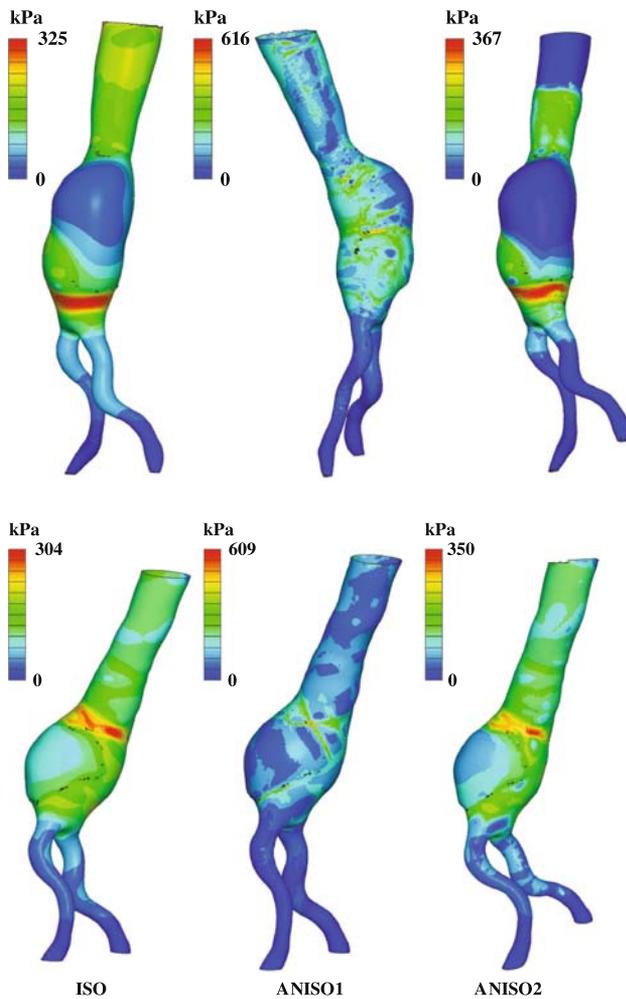
where  $c_{01}$  and  $c_{02}$  are material constants with dimensions of stress, and  $\bar{I}_2$  is the second modified invariant.

The AAA wall material parameters for the isotropic constitutive model (ISO) given by Eq. (1) are obtained from Raghavan and Vorp<sup>11</sup>:  $c_{10} = 174$  kPa and  $c_{20} = 1881$  kPa. For the first anisotropic model (ANISO1), Eq. (2), the parameters are obtained by means of a nonlinear regression analysis of the membrane solution to the experimental data reported by Raghavan *et al.*<sup>12</sup> The regression procedure ( $R^2 = 0.98$ ) yielded:  $c_{10} = 110$  kPa,  $k_1 = k_3 = 210$  kPa,  $k_2 = k_4 = 1700$ , with an orientation angle  $\theta = 43^\circ$  denoting the angle between the fiber reinforcement and the circumferential direction of the wall ( $\cos \theta = \mathbf{m}_0 \cdot \mathbf{e}_\theta = \mathbf{n}_0 \cdot \mathbf{e}_\theta$ ). For the second anisotropic model (ANISO2), Eq. (3), the parameters are obtained by nonlinear fitting of experimental data reported by Vande Geest *et al.*,<sup>15</sup> constraining the fiber orientation to  $\theta = 5^\circ$  according to data reported by Gasser *et al.*<sup>4</sup> on aneurysmal tissue. The regression procedure resulted in the following parameters for the model ( $R^2 = 0.86$ ):  $c_{10} = 0.5$  kPa,  $k_1 = k_3 = 244.9$  kPa,  $k_2 = k_4 = 1576.2$ ,  $\rho = 0.14$ ,  $\bar{I}_4^0 = \bar{I}_6^0 = 1.038$ . This regression was also

performed by allowing the fiber orientation to be a free parameter, yielding ( $R^2 = 0.89$ ):  $c_{10} = 0.5$  kPa,  $k_1 = k_3 = 516.9$  kPa,  $k_2 = k_4 = 1613.0$ ,  $\rho = 0.31$ ,  $\bar{I}_4^0 = \bar{I}_6^0 = 1.066$ , with  $\theta = 21^\circ$ . While the fitting is better, incorporating directly identified structural data by means of independent procedures is more reliable and physically sound. For this reason, we decided to use the set of parameters identified by constraining the fiber orientation to  $\theta = 5^\circ$ . For the ILT, the mean properties reported by Di Martino and Vorp<sup>1</sup> are used in the isotropic model given by Eq. (4):  $c_{01} = 28$  kPa and  $c_{02} = 28.6$  kPa.

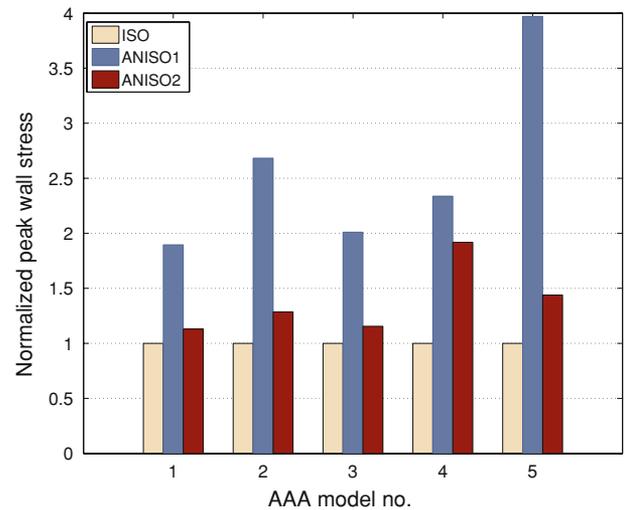
## RESULTS

Triangular surface meshes for the aneurysm wall and three-dimensional quadratic tetrahedral meshes for the thrombus were generated from reconstructions of computed tomography (CT) images of AAA from five subjects treated at Allegheny General Hospital in Pittsburgh, Pennsylvania. For reference, the average maximum diameter of these AAAs is  $67.4 \pm 13.5$  mm. The corresponding DICOM images for all subjects were imported into an in-house Matlab based image segmentation code [VESSEG v.1.0.1, Carnegie Mellon University<sup>9,14</sup>], for the lumen, inner wall, and outer wall segmentations, and the software Simpleware (Simpleware Ltd., Exeter, UK) was employed to create the thrombus and wall meshes. The orientation of collagen fibers was included in the model by defining a tangent at each integration point. The vectors are oriented according to the angle  $\theta$ , with respect to the circumferential direction of the artery. The isotropic and three-dimensional anisotropic material constitutive models given by Eqs. (1)–(3) were implemented within the multi-purpose finite element program ABAQUS by means of user material subroutines (UMAT) and incorporated in the shell element formulation using the algorithm proposed by Klinkel and Govindjee,<sup>8</sup> i.e. condensing an arbitrary 3D material law with respect to the zero-stress condition on which the shell theory is formulated. An intraluminal, static pressure of 120 mmHg was applied on the inner wall surface of the models. Anatomical tethering was simulated by restricting the motion of the artery at the proximal and distal ends (suprarenal inlet and common iliac artery outlet). A tied contact boundary condition was applied at the thrombus–wall interface, resulting in no relative displacement of the thrombus with respect to the wall at this surface. Statistical comparisons of the peak wall stress predicted by the three constitutive laws were performed by means of independent two-tailed *t*-tests at a significance level of  $\alpha = 0.05$ .



**FIGURE 1.** The stress field in two patient specific AAA geometries for the three constitutive laws considered. Note that the ISO and ANISO2 models defined by Eqs. (1) and (3) have a good agreement regarding the location of the maximum principal stress.

Figure 1 shows the stress field in two patient specific models for the three wall constitutive laws considered. The results show that material anisotropy scales up the maximum stress in the aneurysm. However, a good agreement in the location of the maximum stress is obtained between the ISO and ANISO2 models defined by Eqs. (1) and (3). For ANISO1, Eq. (2), the maximum principal stress is larger than that predicted with the other two models and showed poor agreement in the location of the maximum principal stress. Figure 2 illustrates the normalized peak wall stress for all AAAs for each constitutive law implemented. The normalized peak wall stress is calculated as the ratio of the peak wall stress for each constitutive model and the peak wall stress obtained with the isotropic model. There is no significant difference in the mean peak wall stress predicted with ISO and ANISO2 ( $p = 0.129$ ).



**FIGURE 2.** Normalized peak wall stress for five AAA models with the three different constitutive laws. Anisotropy appears to scale up the isotropic peak wall stress, even though results are sensitive to the type of anisotropic material and statistically significant only for ANISO1.

However, the mean peak wall stress predicted by ANISO1 is significantly different from that obtained by ISO ( $p = 0.017$ ) or ANISO2 ( $p = 0.071$ ).

## DISCUSSION

In this investigation, we have corroborated previous findings by Rodriguez *et al.*<sup>13</sup> in that incorporating anisotropy in the material behavior of aneurysmal tissue appears to scale up the maximum principal stress acting on the arterial wall of patient specific AAAs. This is consistent with the outcomes of Vande Geest *et al.*<sup>16</sup> and Martufi *et al.*,<sup>10</sup> where the differences in mean peak wall stress between isotropic and anisotropic models were statistically significant. Our results also demonstrate a considerable sensitivity of this biomechanical determinant of rupture potential to the type of anisotropic model used for the stress analysis. In particular, we found that when the fiber orientation is considered as a variable in the non-linear fitting process of the model to experimental stress–strain data, the value obtained could be significantly different from that observed experimentally. For the ANISO2 model, the fiber orientation was constrained to be along the direction of collagen fibers observed experimentally by Gasser *et al.*<sup>4</sup> when fitting Eq. (3), while the remaining parameters were found by a standard non-linear fitting procedure. For the ANISO1 model, the fiber orientation was left unconstrained during the parameter identification of Eq. (2) to the data reported in Raghavan *et al.*<sup>12</sup> It is important to point out that, when the fiber orientation was constrained during the

optimization process with Eq. (3), i.e., for  $\theta = 5^\circ$ , the correlation between the constitutive model and the experimental values was only  $R^2 = 0.86$ . The material constants obtained for ANISO1 and ANISO2 indicate only a minor anisotropy [ $\theta = 43^\circ$  for Eq. (2) and  $\rho = 0.14$  for Eq. (3)]; however, the difference in their predicted mean peak wall stresses is statistically significant. In this regard, for ANISO2, the location of the maximum principal stress correlates well with ISO and the larger anisotropic stress represents a statistically insignificant difference.

These results suggest that anisotropic soft tissue constitutive models should be treated with care when conducting stress analysis on patient specific AAA models, as the results could vary significantly depending on the strain energy function used. Therefore, it is recommended that structural parameters associated with the constitutive law be determined independently by means of physiologic characterization of the tissue structure and avoid their identification through nonlinear fitting to *ex vivo* stress-strain test data.

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